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A Configuration-Oriented SPICE Model for Multiconductor Transmission Lines with Homogeneous Dielectrics

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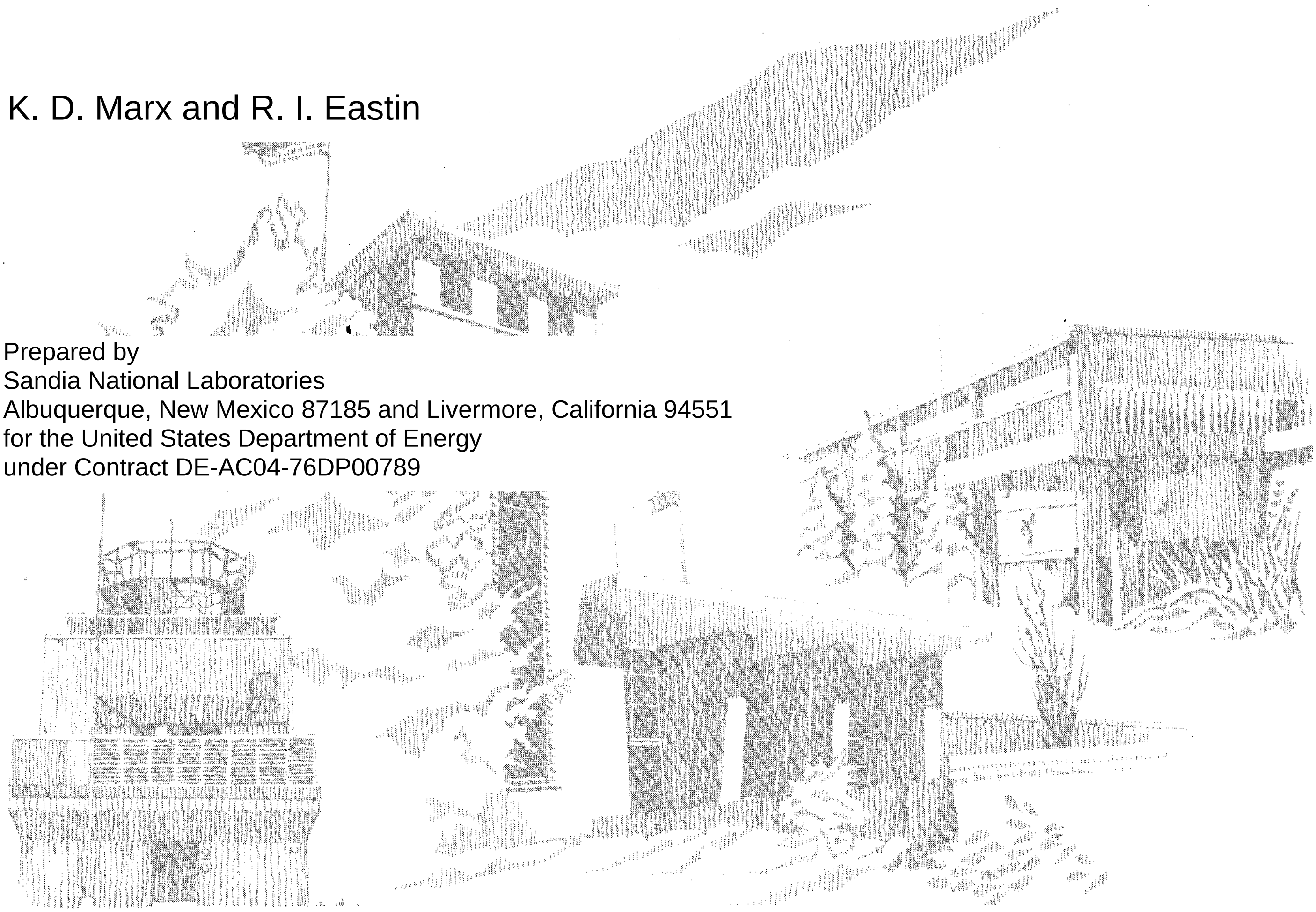

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A Configuration-Oriented SPICE Model for Multiconductor Transmission Lines with Homogeneous Dielectrics

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ABSTRACT

The use of the SPICE circuit analysis computer program to simulate a lossless multiconductor transmission line is investigated. It is demonstrated that for the case of a homogeneous dielectric, the multiconductor line can be represented by a system of standard two-wire lines which is not based on modal decomposition. This system is readily modeled with SPICE. While restricted to situations where the dielectric constant can be assumed homogeneous, the present method has the advantage of an intuitive relationship to the conductor configuration, simpler SPICE input data requirements, and an improvement in computer run time over other methods.

I. INTRODUCTION

It is often necessary to consider systems of interconnecting wires or other conductors as multiconductor transmission lines (MTLs) [1] - [9]. In many cases, the complexity of such systems dictates the use of numerical methods to simulate their electrical response. The SPICE computer program [10] - [13] is often used for this purpose [6] - [9].

The analysis of an MTL with inhomogeneous dielectric requires the decomposition of signals into normal modes of propagation. (On a three-conductor line these modes are often referred to as "even mode" and "odd mode" (for symmetric lines) or "common mode" and "signal mode" [1], [3].) Such modes propagate at different velocities in general. The resulting SPICE models incorporate this decomposition via the introduction of a system of two-conductor transmission lines connected to a network of dependent sources. This procedure is described in References [6] - [9].

The purpose of the present paper is to introduce a simplified SPICE model for MTLs. Our method has two important physical restrictions. First, significant signal propagation is restricted to situations in which the propagation velocity on the line is unique. In principle, this means that the dielectric constant and magnetic permeability must be uniform over that cross-section of the line in which a nonzero electromagnetic field exists [3]. In practice, it is often a good approximation for signals of interest, even on MTLs with inhomogeneous dielectrics. The second restriction is that the lines are assumed to be lossless. It is difficult to relieve this constraint within the framework of the SPICE lossless delay line model. Although not implemented numerically there, References [8] and [9] contain discussions of the problem of lossy MTLs.

It is not the intent of the present paper to be critical in any way of the computational models in References [6] - [9], which deal with the general case of inhomogeneous dielectrics, or equivalently, nonunique modal propagation velocities. The normal mode decomposition is necessary in that case. If, however, one can accept the restriction to a single propagation velocity, our method offers some advantages: (1) The SPICE data files are simpler, and the two-conductor lines implemented in them have an interpretation which is more physically intuitive than that corresponding to normal modes, (2) Experimental measurements on an MTL can be easily converted into the parameters which define the SPICE delay lines, and (3) Computer run times can be reduced. (We note, however, that our procedure may lead to more severe computer memory requirements in some versions of SPICE. This is discussed in Section V.)

In the following section, we give the telegrapher's equations for lossless MTL systems and make a plausible argument indicating that a network of two-wire delay lines can be used to represent an MTL with homogeneous dielectrics. In Section III, a complete proof is given of the equivalence of this network and the original MTL. In Section IV, the resulting model is summarized and methods of determining its parameters are described. In Section V, the method is applied to the problem discussed by Paul [8]. The final section of the paper offers some concluding remarks.

In this work, we deal primarily with time domain analysis. However, the formulation applies equally well to the frequency domain, and this is exploited in the proof in Section III.

II. DERIVATION OF THE MODEL. PRELIMINARIES

If the signal frequencies are sufficiently low that wavelengths are long compared to interconductor spacing, the only propagation modes permitted on an MTL will be TEM or quasi-TEM modes. (If the dielectric constant is inhomogeneous, the modes are referred to as quasi-TEM, because then it is in general impossible for the electric and magnetic fields to be literally transverse [14], [15]). Let the number of conductors, including the ground or return line, be *n*. (See Fig. 1.) Propagation on the line is described by the telegrapher's equations [1] - [9].

$$\frac{\partial \boldsymbol{V}(z,t)}{\partial z} = -\boldsymbol{L}\frac{\partial \boldsymbol{I}(z,t)}{\partial t} \tag{1}$$

$$\frac{\partial \boldsymbol{I}(z,t)}{\partial z} = -\boldsymbol{C}\frac{\partial \boldsymbol{V}(z,t)}{\partial t} \tag{2}$$

where $\boldsymbol{V}(z,t)$ and $\boldsymbol{I}(z,t)$ are $(n-1)$- dimensional column vectors which represent the voltage and current on the conductors, and $\boldsymbol{L}$ and $\boldsymbol{C}$ are $(n-1) \times (n-1)$ inductance and capacitance (per unit length) matrices. If the dielectric is homogeneous, we have [3], [8]

$$\boldsymbol{L} = \frac{1}{v_o^2}\boldsymbol{C}^{-1} \tag{3}$$

where v_o is the propagation velocity in the dielectric, given by

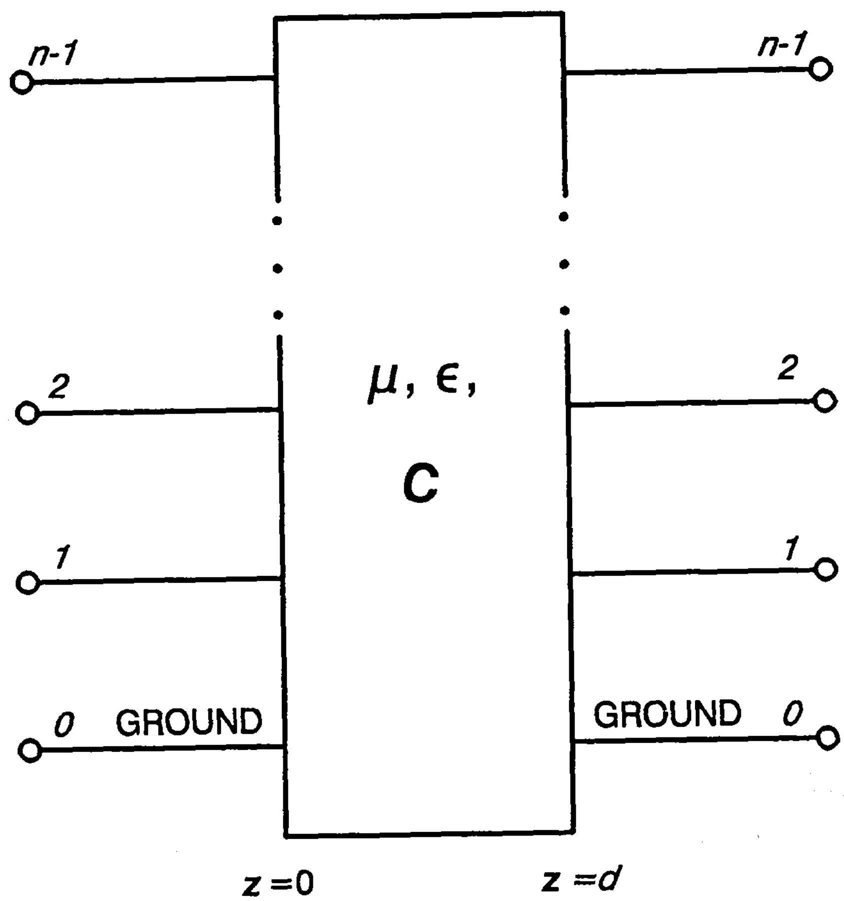

Figure 1. Schematic of an n-conductor transmission line with homogeneous dielectric. The line is completely characterized by the permeability μ, dielectric constant ε, capacitance matrix **C** and length d.

$$v_o = \frac{1}{\sqrt{\mu\varepsilon}} \tag{4}$$

and μ and ε are the magnetic permeability and dielectric constant, respectively.

It may further be shown [3] that for this case, signals on the line consist of waves which propagate in the forward and backward directions on the line with velocity v_o. For these waves, the current and voltage are related by

$$\boldsymbol{I_f} = \boldsymbol{Y_o}\,\boldsymbol{V_f} \tag{5}$$

$$\boldsymbol{I_b} = -\,\boldsymbol{Y_o}\,\boldsymbol{V_b} \tag{6}$$

where subscripts f and b refer to forward- and backward-travelling waves, respectively, and $\boldsymbol{Y_o}$ is the characteristic admittance matrix, given by

$$\boldsymbol{Y_o} = v_o\,\boldsymbol{C} \tag{7}$$

We will now present a system of two-wire lines that is an exact synthesis of the MTL shown in Fig. 1. Proof that the synthesis is correct will be given in the following section.

To derive the model, we imagine that we have an infinitely long MTL. Then any n-port source connected to it sees a purely resistive n-port network with admittance matrix $\boldsymbol{Y_o}$ [3]. This suggests that one can simulate the MTL with a system of two-wire lines, one for each pair of ports with characteristic impedances chosen so that the admittance matrix is just $\boldsymbol{Y_o}$. This arrangement is shown schematically for a 4-conductor line in Fig. 2.

In general, the number m of two-wire lines required will be

$$m = \frac{n(n-1)}{2} \tag{8}$$

The line connecting conductor α to conductor β will be designated $T_{\alpha\beta}$, with characteristic impedance $z_{o\alpha\beta}$. (Ground is conductor 0.)

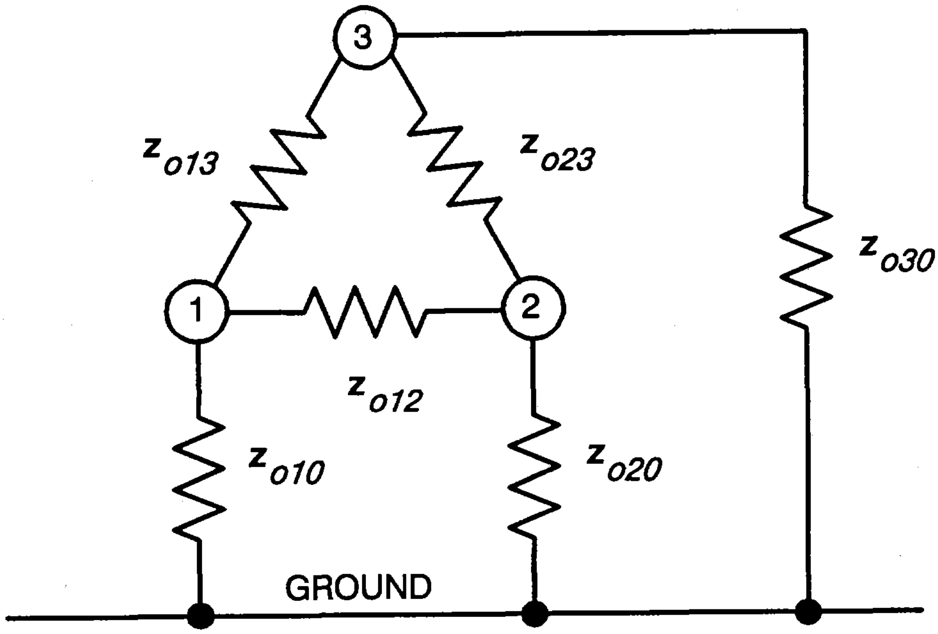

Figure 2. Representation of the synthesis of a 4-conductor transmission line with 6 two-wire lines. An end-on view of 3 round wires over a ground plane is shown. The resistance $z_{o\alpha\beta}$ represents the characteristic impedance of the two-wire line connected between conductor α and conductor β

It is easy to show that it is possible to select the $z_{o\alpha\beta}$ so that the admittance matrix is $\boldsymbol{Y_o}$. The matrix has the following properties [3], [16]. It is real, symmetric, dominant, and has positive diagonal elements and negative off-diagonal elements. Let the elements be $Y_{o\alpha\beta}$. By definition, $Y_{o\alpha\beta}$ is the current flowing into conductor α per unit voltage applied to conductor β with all conductors other than β shorted to ground.* The network of $z_{o\alpha\beta}$s will satisfy these requirements if we choose

$$z_{o\alpha\beta} = -\frac{1}{Y_{o\alpha\beta}} \quad , \qquad \alpha \neq \beta,\ \beta \neq 0 \qquad (9)$$

$$z_{o\alpha 0} = \frac{1}{\sum_{\beta=1}^{n-1} Y_{o\alpha\beta}} \qquad (10)$$

where the sum is over all conductors including the αth conductor. Recall that $Y_{o\alpha\beta}$ is real and negative if $\alpha \neq \beta$. Then, as evaluated in (9), $z_{o\alpha\beta}$ is real and positive, and can be realized with an ideal delay line. Furthermore, $Y_{o\alpha\alpha}$ is positive and $\boldsymbol{Y_o}$ is dominant. Hence the sum in (10) is positive, and $z_{o\alpha 0}$ is similarly realizable.

---

*The following may aid in an intuitive interpretation of $\boldsymbol{Y_o}$: Any diagonal element $Y_{o\alpha\alpha}$ is positive because it is the current into conductor α per unit voltage applied to the *same* conductor. Conversely, when all conductors except the α th are grounded, their currents are negative (or zero) with respect to the sign of the voltage on conductor α, since they each carry (at most) part of the return current. Hence, $Y_{o\alpha\beta} < 0 \ \alpha \neq \beta$. Furthermore, the magnitude of the sum of all these return currents must be less than the input current (current on the ground conductor is not included in this sum). This means that the matrix is dominant:

$$Y_{o\alpha\alpha} > -\sum_{\alpha \neq \beta} Y_{o\alpha\beta}$$

Finally, the matrix is symmetric by the usual reciprocity arguments, and it is real because of the nature of TEM waves.

This result for an MTL of infinite length *suggests* that a *finite-length* MTL with homogeneous dielectric can be simulated with m two-wire lines of the same length and propagation velocity, where m is given by (8), and the characteristic impedances are given by (9) and (10). In the following section, we give a complete proof for the finite-length line. (Note: Section III can be skipped without loss of continuity.)

III. PROOF THAT THE SYSTEM OF TWO-WIRE LINES SIMULATES THE MTL

The desired proof appears to be most easily obtained by considering the frequency domain. The temporal factor $e^{j\omega t}$ will be assumed. The general solution of Eqs. (1) and (2) is then [17]

$$\boldsymbol{V} = \boldsymbol{A}\, e^{-jkz} + \boldsymbol{B}\, e^{jkz}$$

$$\boldsymbol{I} = \boldsymbol{Y_0}(\boldsymbol{A}\, e^{-jkz} - \boldsymbol{B}\, e^{jkz})$$

where

$$k = \frac{\omega}{v_0}$$

is the wave number on the line, and $\boldsymbol{A}$ and $\boldsymbol{B}$ (the amplitudes of the forward and backward waves) are arbitrary vectors (up to this point). We wish to consider the n-conductor MTL of Fig. 1 as a $2n$-port network. This means that we want to obtain relationships between all the voltages and currents at both ends of the line. Subscripts a and b will be used to signify quantities at $z=0$ and $z=d$, respectively. Then the quantities of concern are

$$\boldsymbol{V_a} = \boldsymbol{A} + \boldsymbol{B}$$

$$\boldsymbol{I_a} = \boldsymbol{Y_0}\,(\boldsymbol{A} - \boldsymbol{B})$$

$$V_b = A\, e^{-jkd} + B\, e^{jkd}$$

$$I_b = Y_o\, (A\, e^{-jkd} - B\, e^{jkd})$$

Eliminating ***A*** and ***B*** and rearranging, we obtain

$$-j \sin kd\, I_a = Y_o\, (V_b - \cos kd\, V_a) \tag{11}$$

$$I_b - \cos kd\, I_a = -j \sin kd\, Y_o\, V_a \tag{12}$$

Now consider the general network of $n(n-1)/2$ two-wire lines shown in Fig. 3. We wish to show that the signals at the terminals of this network will satisfy Eqs. (11) and (12) if the characteristic impedances are given by Eqs. (9) and (10). Lower-case symbols will be used to denote the voltages and currents on these delay lines; i.e., $v_{\alpha\beta}$ and $i_{\alpha\beta}$ are the voltages and currents on line $T_{\alpha\beta}$, which is connected between nodes α and β. It is to be understood that the delay times for all the $T_{\alpha\beta}$ are the same as the delay time of the MTL:

$$t_d = \frac{d}{v_o} \tag{13}$$

Since Eqs. (11) and (12) hold equally well for a simple two-wire line, we have

$$-j \sin kd\, i_{a\alpha\beta} = y^t_{o\alpha\beta}\, (v_{b\alpha\beta} - \cos kd\, v_{a\alpha\beta}) \tag{14}$$

$$i_{b\alpha\beta} - \cos kd\, i_{a\alpha\beta} = -j \sin kd\, y^t_{o\alpha\beta}\, v_{a\alpha\beta} \tag{15}$$

for any α, β, where

$$y^t_{o\alpha\beta} = \frac{1}{z_{o\alpha\beta}} \tag{16}$$

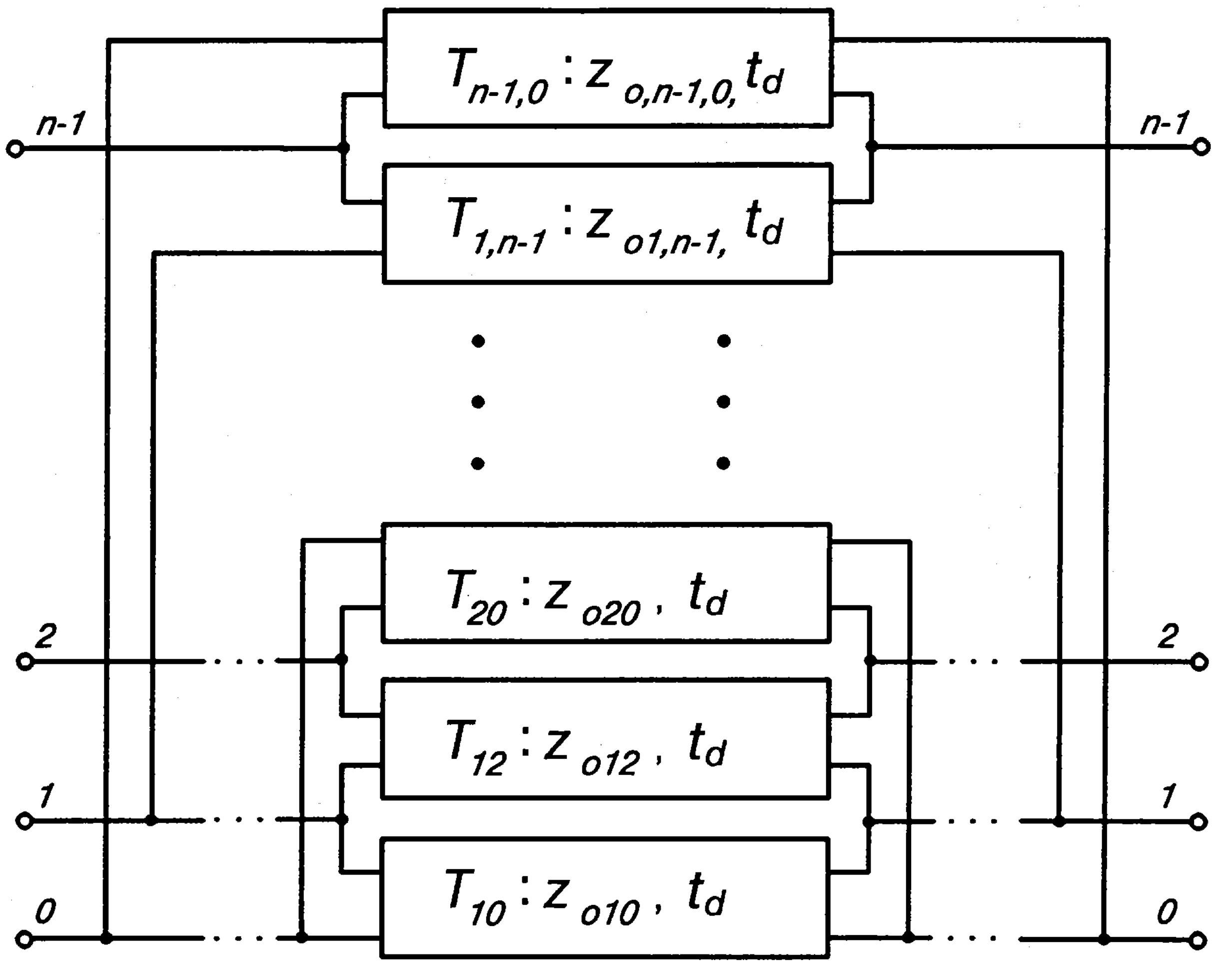

Figure 3. Network of *n(n-1)/2* two-wire lines which simulates the general *n*-conductor MTL shown in Fig. 1.

is the characteristic admittance of $T_{\alpha\beta}$ (superscript "t" for "two-wire").

In the model of Fig. 3, the indices are to be ordered so that the two-wire line voltages are related to the node voltages on the MTL by

$$v_{\alpha\beta} = \begin{cases} V_\alpha - V_\beta & \alpha > \beta \\ V_\beta - V_\alpha & \alpha < \beta \end{cases} \tag{17}$$

In other words, the reference node on the two-wire line corresponds to the second index if it is lower than the first, and vice versa. This choice is arbitrary, but it must be observed systematically in what follows. Then the simulated MTL currents are

$$I_\alpha = i_{\alpha 0} + i_{\alpha 1} + \dots + i_{\alpha,\alpha-1} - i_{\alpha,\alpha+1} - \dots - i_{\alpha,n-1} \tag{18}$$

Note that we will subsequently be appending subscripts a and b to the terms in Eqs. (17) and (18) to denote signals at $z=0$ and $z=d$.

From (14) and (18), we obtain a relation between the simulated MTL currents I_a (at $z=0$) and the voltages $v_{a\alpha\beta}$ and $v_{b\alpha\beta}$:

$$\begin{aligned} -j \sin kd\, I_{a\alpha} = {} & y^t_{o\alpha 0} (v_{b\alpha 0} - \cos kd\, v_{a\alpha 0}) \\ & + y^t_{o\alpha 1} (v_{b\alpha 1} - \cos kd\, v_{a\alpha 1}) + \dots \\ & + y^t_{o\alpha,\alpha-1} (v_{b\alpha,\alpha-1} - \cos kd\, v_{a\alpha,\alpha-1}) \\ & - y^t_{o\alpha,\alpha+1} (v_{b\alpha,\alpha+1} - \cos kd\, v_{a\alpha,\alpha+1}) - \dots \\ & - y^t_{o\alpha,n-1} (v_{b\alpha,n-1} - \cos kd\, v_{a\alpha,n-1}) \end{aligned} \tag{19}$$

Using (17), we convert (19) into a formula involving the simulated MTL currents and node voltages:

$$-j \sin kd \, I_{a\alpha} = \sum_{\beta=0}^{n-1}{}' \, y^{t}_{o\alpha\beta} \, (V_{b\alpha} - \cos kd \, V_{a\alpha})$$

$$- \sum_{\beta=1}^{n-1}{}' \, y^{t}_{o\alpha\beta} \, (V_{b\beta} - \cos kd \, V_{a\beta}) \tag{20}$$

where $\Sigma\,'$ means to sum over all values of β not equal to α.

By manipulating (15), (17), and (18) in exactly the same way, we obtain

$$I_{b\alpha} - \cos kd \, I_{a\alpha} - j \sin kd \left(\sum_{\beta=0}^{n-1}{}' \, y^{t}_{o\alpha\beta} \, V_{a\alpha} - \sum_{\beta=1}^{n-1}{}' \, y^{t}_{o\alpha\beta} \, V_{a\beta} \right) \tag{21}$$

We see that (20) and (21) are exactly equivalent to the α th components of (11) and (12) if we choose

$$Y_{o\alpha\alpha} = \sum_{\beta=0}^{n-1}{}' \, y^{t}_{o\alpha\beta} \tag{22}$$

$$Y_{o\alpha\beta} = -\, y^{t}_{o\alpha\beta} \quad , \quad \alpha \neq \beta \tag{23}$$

But (23) is the same as (9) (in view of (16)). Furthermore, upon application of (23), Eq. (22) becomes

$$Y_{o\alpha\alpha} = y^{t}_{o\alpha 0} - \sum_{\beta=1}^{n-1}{}' Y_{o\alpha\beta}$$

so that

$$z_{o\alpha 0} = \frac{1}{y^{t}_{o\alpha 0}} = \frac{1}{\sum_{\beta=1}^{n-1} Y_{o\alpha\beta}}$$

which is identical to (10).

Therefore, the network shown in Fig. 3 is completely equivalent to the MTL of Fig. 1 if (9), (10), and (13) are satisfied. Furthermore, from the results of Section II, they can always be satisfied by a network of lossless two-wire lines. This completes the proof. The network in Fig. 3 can easily be modeled by a system of SPICE delay lines once the $z_{o\alpha\beta}$ and t_d are known.

IV. SUMMARY OF THE MODEL. COMMENTS ON DETERMINATION OF THE PARAMETERS

In summary, our SPICE model for an MTL is as follows: set up a network of two-wire delay lines wherein there is one delay line connecting each pair of conductors in the MTL, as shown in Fig. 3. The relevant equations describing this configuration are collected in Table I for reference.

Before proceeding to a specific application, we offer some comments on the determination of the values of $z_{o\alpha\beta}$ required for the model. From Eqs. (9) and (10), we see that this is tatamount to determining $\boldsymbol{Y_o}$. Furthermore, from (7) it is clear that if we know v_o, knowledge of the components of $\boldsymbol{C}$ suffices. Theoretical methods of determining capacitance coefficients are described in many textbooks on electromagnetic theory [17] - [20]. Complex geometries can be treated via numerical solutions to Laplace's equation. We will not describe these methods further. However, in what follows, we will discuss some relationships between matrix elements and experimental methods for determining them.

Table I. Parameters defining two-wire delay lines in the SPICE model. (See Fig. 3)

Number of conductors in MTL	n	
Number of interconnecting delay lines in SPICE model	$m = \frac{n(n-1)}{2}$	(8)
Characteristic impedances of delay lines (in terms of elements of characteristic admittance matrix)	$z_{o\alpha\beta} = \frac{-1}{Y_{o\alpha\beta}}$, $\alpha \neq \beta, \beta \neq 0$	(9)
	$z_{o\alpha 0} = \frac{1}{\sum_{\beta=1}^{n-1} Y_{o\alpha\beta}}$	(10)
Delay time (same as for MTL)	$t_d = \frac{d}{v_o}$	(13)

The instrumentation that one commonly has available consists of a time domain reflectometer (TDR) and/or an impedance analyzer with which the characteristic impedance and/or the capacitance can be measured between *two* terminals. We have found it convenient to use both of these instruments, one being a check against the other. The TDR can also be used to measure the time delay t_d for the system. Furthermore, it can provide information on the propagation velocities of different modes in a system with inhomogeneous dielectrics. Hence, it can help determine whether the system can be treated approximately as an MTL with homogeneous dielectrics in a particular situation.

Using a standard TDR or impedance analyzer, one can readily measure the diagonal terms $Y_{o\alpha\alpha}$ or $C_{\alpha\alpha}$. However, the off-diagonal terms $Y_{o\alpha\beta}$ or $C_{\alpha\beta}$, $\alpha \neq \beta$, are not directly measurable. To obtain them, one must make a series of measurements between each pair of ports and then solve a system of equations for the elements of $\boldsymbol{Y_o}$ or $\boldsymbol{C}$.

One systematic way to do this is to obtain the inverse matrix by measuring the admittance or capacitance between each pair of terminals with all other terminals open-circuited. We will use the admittance matrix as an example, but the capacitance matrix can be treated in a completely analogous manner. The inverse of the admittance matrix is the characteristic impedance matrix:

$$\boldsymbol{Z_o} = \boldsymbol{Y_o}^{-1} \tag{24}$$

The component $Z_{o\alpha\beta}$ of $\boldsymbol{Z_o}$ is the voltage on the αth conductor per unit current into the βth conductor with all conductors other than the βth (and ground) open-circuited. Now consider measuring the impedance between each pair of terminals in the MTL with all other terminals open-circuited. Let the results of those measurements be $z^m_{o\alpha\beta}$ ($\alpha,\beta=0,1,\ldots,n-1$). By definition, the diagonal elements of $\boldsymbol{Z_o}$ are given by

$$Z_{o\alpha\alpha} = z^m_{o\alpha o} \tag{25}$$

To determine the off-diagonal elements $Z_{o\alpha\beta}$, $\alpha \neq \beta$, we note that the process of measuring $z^m_{o\alpha\beta}$ implies application of the formula

$$z^m_{o\alpha\beta} = \frac{V_\alpha - V_\beta}{I_\alpha} \tag{26}$$

where V_α and V_β are voltages referenced to ground. Since all terminals other than α and β are open-circuited, $I_\beta = -I_\alpha$. From the definition of $\mathbf{Z_o}$, we then have

$$\begin{aligned} V_\alpha &= Z_{o\alpha\alpha}\, I_\alpha + Z_{o\alpha\beta}\, I_\beta \\ &= (Z_{o\alpha\alpha} - Z_{o\alpha\beta})\, I_\alpha \\ V_\beta &= Z_{o\beta\alpha}\, I_\alpha + Z_{o\beta\beta}\, I_\beta \\ &= (Z_{o\alpha\beta} - Z_{o\beta\beta})\, I_\alpha \end{aligned}$$

where we have used the reciprocity relation $Z_{o\beta\alpha} = Z_{o\alpha\beta}$. Substituting these expressions into (26), using (25) to eliminate $Z_{o\alpha\alpha}$ and $Z_{o\beta\beta}$, and rearranging, we obtain

$$Z_{o\alpha\beta} = \frac{1}{2}(z^m_{o\alpha 0} + z^m_{o\beta 0} - z^m_{o\alpha\beta}), \qquad \alpha \neq \beta \tag{27}$$

Once the elements of $\mathbf{Z_o}$ are determined, the elements of $\mathbf{Y_o}$ can be obtained by matrix inversion. Determination of these parameters is simpler than is the case for modal decomposition in that only matrix inversion is required, rather than solution to an eigenvalue problem.

It may be noted that, in view of Eqs. (3) and (24), a measurement of the inductance matrix yields the impedance matrix $\mathbf{Z_o}$. However, inductance measurements are sometimes more ambiguous than capacitance measurements because of the internal inductance and resistive losses of the conductors, and the frequency dependence of these parameters. Hence, we have chosen to ignore them in this discussion. Capacitance measurements made at low frequency will yield the admittances for the ideal case of a lossless line.

V. APPLICATION

As an example of the application of the model in a SPICE simulation, we consider the system studied by Paul [8]. The configuration is shown in Fig. 4. The dielectric constant and permeability are those of vacuum ($v_o = 3 \times 10^8$ m/s) and the length of the line is $d = 4.67$ m. Hence, from (13), the time delay is $t_d =$ 15.58 ns. Paul gives the inductance matrix as

$$\boldsymbol{L} = \begin{pmatrix} 0.9179 & 0.1609 \\ 0.1609 & 0.9179 \end{pmatrix} \quad (\mu\text{H}) \tag{28}$$

(Note that the MTL is symmetric with respect to conductors 1 and 2.)

We could simply evaluate $\boldsymbol{Y_o}$ by inverting $\boldsymbol{L}$ to obtain $\boldsymbol{C}$ (see Eq. (3)), and then applying (7). However, to illustrate the experimental considerations discussed in the previous section, we will take the point of view that we are determining the parameters experimentally. Consider the MTL as shown in Fig. 4 (a). If one connects a TDR from ground to either conductor 1 or conductor 2, leaving the other open-circuited, an impedance of

$$z^m_{o10} = z^m_{o20} = 275.3 \text{ ohm}$$

will be measured. The impedance between conductors 1 and 2 (with ground open) will be

$$z^m_{o12} = 454.1 \text{ ohm}$$

(In this case, these values were obtained from appropriate manipulation of the inductance matrix; they are the theoretical values of the measured impedances.)

Then the elements of the characteristic impedance matrix are given by (25) and (27) as

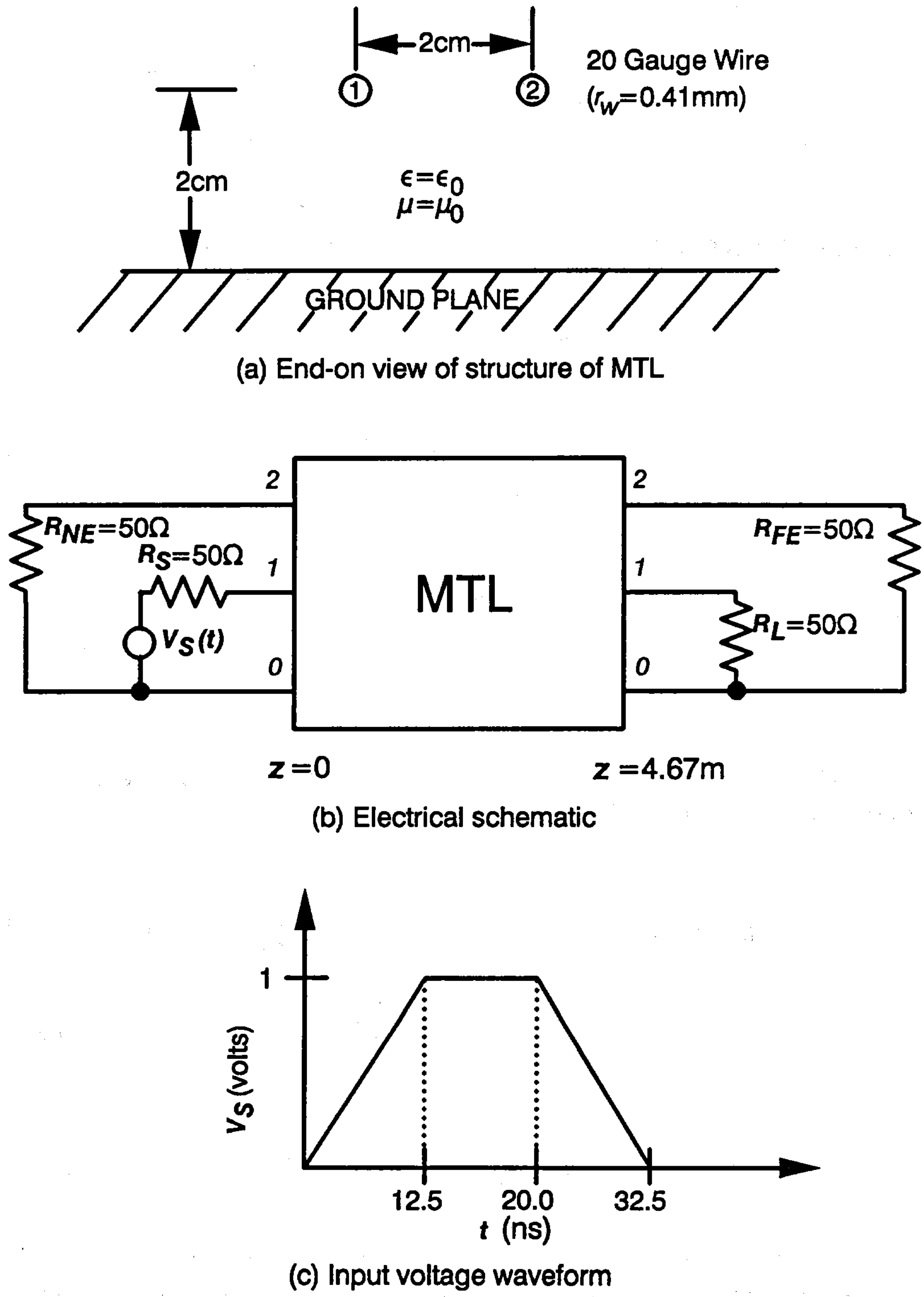

Figure 4. Three-conductor line and electrical configuration used for the sample problem. (From Paul [8].)

$$Z_{o11} = Z_{o22} = 275.3 \text{ ohm}$$

$$Z_{o12} = \frac{1}{2}(275.3 + 275.3 - 454.1)$$

$$= 48.2 \text{ ohm}$$

The characteristic admittance matrix is obtained from (24):

$$Y_o = Z_o^{-1}$$

$$= \begin{pmatrix} 275.3 & 48.2 \\ 48.2 & 275.3 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 3.747 & -0.657 \\ -0.657 & 3.747 \end{pmatrix} \text{ (mmho)}$$

Then Eqs. (9) and (10) give the values of the two-wire characteristic impedances for the SPICE model:

$$z_{o10} = z_{o20} = \frac{10^3}{3.747 - 0.657} = 323.6 \text{ ohm}$$

$$z_{o12} = \frac{-10^3}{-0.657} = 1522 \text{ ohm}$$

The resulting SPICE network is shown in Figure 5. The input data file is shown in Table II.

The input data file for Paul's modal decomposition method is reproduced from Reference 8 in Table III. The present method employs three two-wire lines as opposed to two in Paul's method. However, it is clear that the use of two-wire lines instead of the modal decomposition method leads to considerable simplification.

We ran SPICE with both these models. We used two different versions: PSPICE on an IBM PS/2 Model 80, and SPICE Version 2G on a VAX-8650

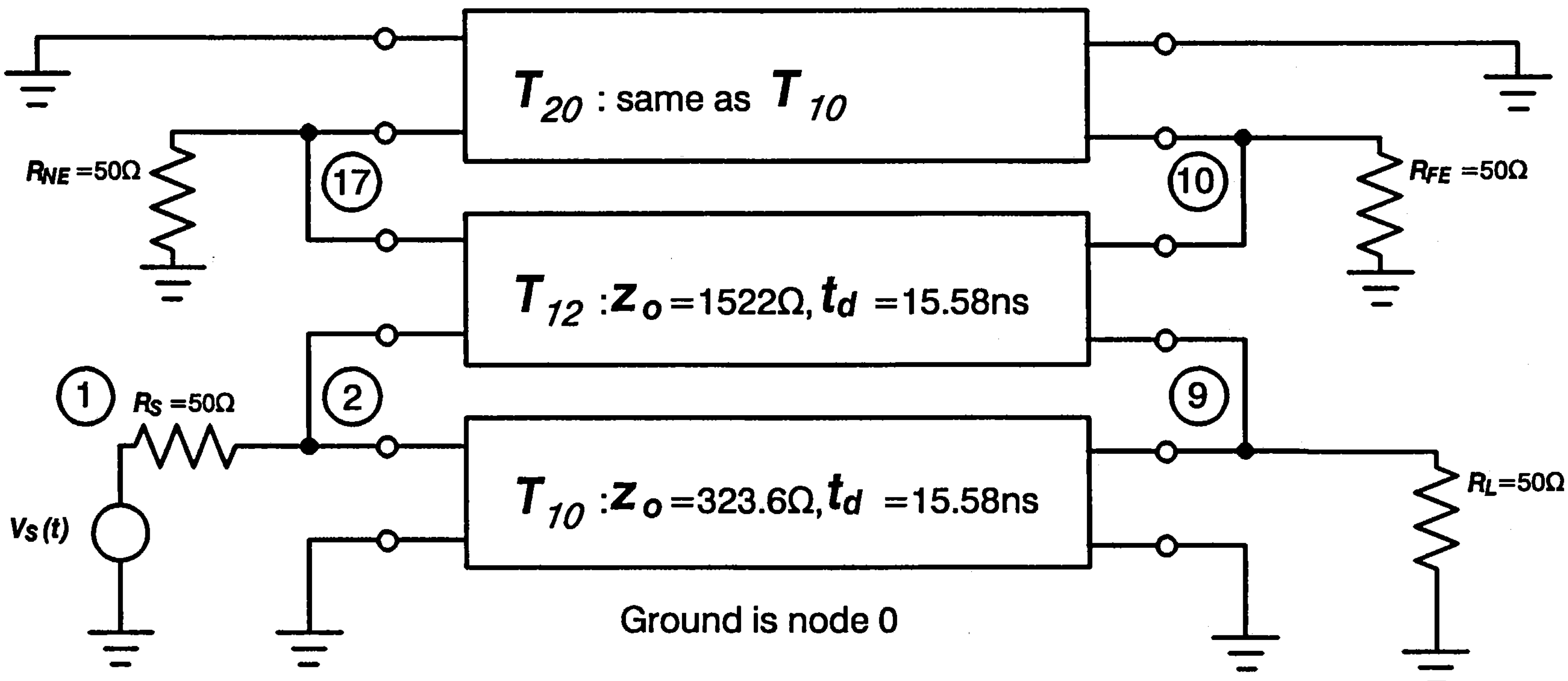

Figure 5. The SPICE network resulting from the application of the present model to the sample problem used by Paul [8]. The numbers in circles identify the nodes used in Table II. (The corresponding nodes also appear in Table III.)

Table II. SPICE input data file for the present method. (Time is expressed in nanoseconds.)

```
* MTL MODEL USING TWO-WIRE DELAY LINES
*
VS    1  0   PULSE (0 1 0 12.5 12.5 7.5 1000)
RS    1  2   50
RL    9  0   50
RFE 10  0   50
RNE 17  0   50
*
T01   2  0   9  0 ZO=323.6 TD=15.58
T12   2 17   9 10 ZO=1522   TD=15.58
T02 17  0 10  0 ZO=323.6 TD=15.58
*
.TRAN      .5  200.
.OPTIONS  LIMPTS=801 PIVTOL=1.0E-16
.PRINT TRAN V(1) V(2) V(9) V(17) V(10)
.END
```

Table III. SPICE input data file for Paul's method.
(From Reference 8.) (Time is expressed in nanoseconds.)

```
* MTL MODEL USING MODAL DECOMPOSITION
*
VS    1    0    PULSE    (0 1 0 12.5 12.5 7.5 1000)
RS    1    2    50
V1    2    3
RL    9    0    50
V3    9    8
RFE  10    0    50
V4   10   11
RNE  17    0    50
V2   17   16
E1    3    4     5    0     .707
E2    4    0    14    0    -.707
E3    8    7     6    0     .707
E4    7    0    13    0    -.707
E5   11   12     6    0     .707
E6   12    0    13    0     .707
E7   16   15     5    0     .707
E8   15    0    14    0     .707
F1    0    5    V1     .707
F2    0    5    V2     .707
F3    0    6    V3     .707
F4    0    6    V4     .707
F5    0   13    V3    -.707
F6    0   13    V4     .707
F7    0   14    V1    -.707
F8    0   14    V2     .707
T1    5    0     6    0      Z0=323.70  TD=15.58
T2   14    0    13    0      Z0=227.07  TD=15.58
.TRAN      .5   200
.OPTIONS        LIMPTS=801
.PRINT TRAN V(1) V(2) V(9) V(17) V(10)
.END
```

computer running the VMS operating system. The results from the two models agree to essentially four significant figures on either computer; voltage across resistor R_{NE} is shown in Fig. 6.

We found that the present model used approximately 33% less CPU time than that required for Paul's model for this problem when run with PSPICE. There was an even greater difference of 66% less on the VAX. (These figures include cases in which the problem time was extended in order to obtain better CPU statistics.) Hence, it appears that the present method can also offer advantages of computational efficiency.

It should be noted that the delay line model in some versions of SPICE may use relatively large amounts of computer memory [11]. The number of delay lines required to simulate an n-conductor line using our model is $m = n(n-1)/2$, compared to $n-1$ delay lines for a modal decomposition model. Hence, a point could be reached where one would have to use a modal decomposition model to avoid running out of memory. We have, in fact, encountered memory limitations when running simulations requiring five delay lines with our model on the VAX version of SPICE. The effect of this is to restrict the physical simulation time available. However, so far we have not had this difficulty when running PSPICE. In particular, the same problems that failed on the VAX version ran without incident on PSPICE. Hence, many practical problems are readily accessible to the model.

VI. CONCLUDING REMARKS

This paper has presented a method for simulating multiconductor transmission lines with the SPICE circuit analysis program. The method offers an alternative to the modal decomposition methods proposed by other authors. It should be repeated here that this work does not intend to offer criticism of the other methods. Modal decomposition is necessary when it is important to consider different propagation velocities on lines with inhomogeneous dielectrics. Furthermore, in some cases the modal decomposition methods might use less computer memory. However, we have found that our method leads to simpler SPICE input data structures (hence, shorter setup times) and reduced CPU time. The simpler input data is actually a result of a very intuitive physical interpretation of the wire-to-wire characteristic impedances measured on the MTL. This interpretation has been emphasized above in a discussion of experimental and theoretical procedures for determining the parameters used in SPICE calculations.

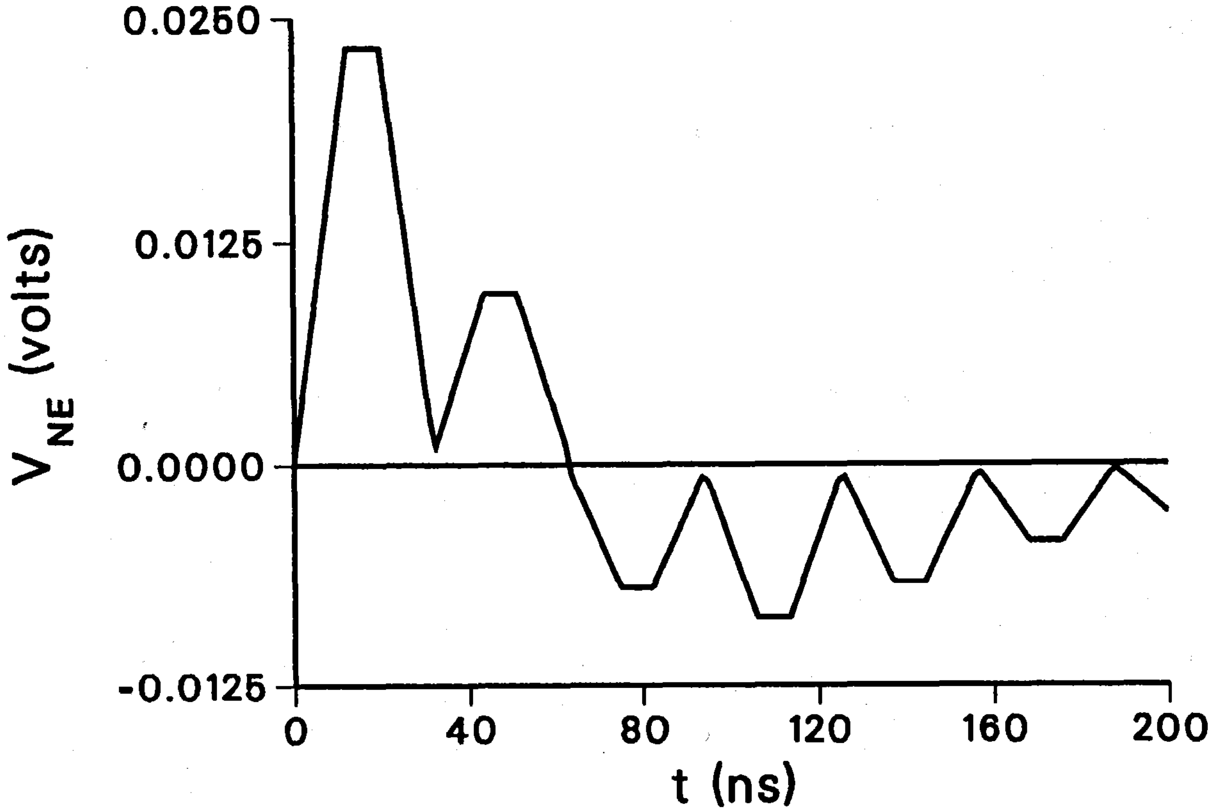

Figure 6. Output from SPICE: Voltage waveform across resistor R_{NE}.

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